

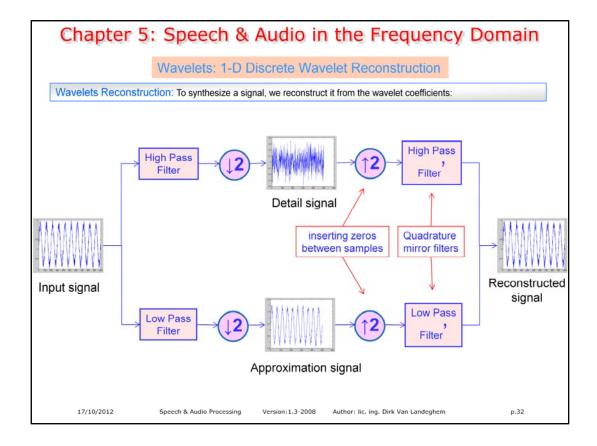
For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavor or nuance. Consider the human voice. If you remove the high-frequency components, the voice sounds different, but you can still tell what's being said. However, if you remove enough of the low-frequency components, you hear gibberish.

In wavelet analysis, we often speak of approximations and details. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components.

The filtering process, at its most basic level is shown above.

The original signal, S, passes through two complementary filters and emerges as two signals. Unfortunately, if we actually perform this operation on a real digital signal, we wind up with twice as much data as we started with. Suppose, for instance, that the original signal S consists of 1000 samples of data. Then the resulting signals will each have 1000 samples, for a total of 2000.

There exists a more subtle way to perform the decomposition using wavelets. By looking carefully at the computation, we may keep only one point out of two in each of the two 2000-length samples to get the complete information. This is the notion of downsampling.



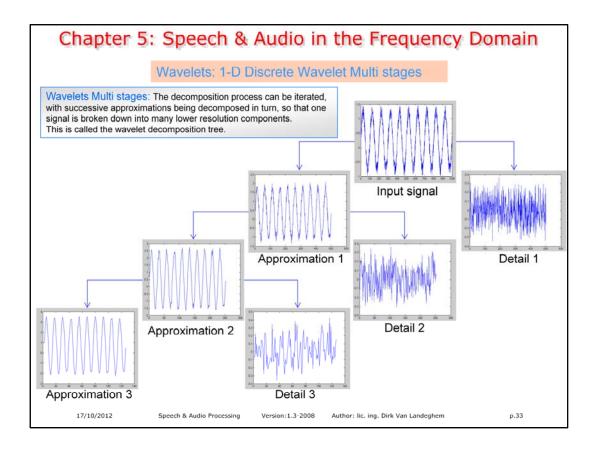
We've learned how the discrete wavelet transform can be used to analyze, or decompose, signals and images. This process is called decomposition or analysis. The other half of the story is how those components can be assembled back into the original signal without loss of information. This process is called reconstruction, or synthesis. The mathematical manipulation that effects synthesis is called the inverse discrete wavelet transform (IDWT).

Where wavelet analysis involves filtering and downsampling, the wavelet reconstruction process consists of upsampling and filtering. Upsampling is the process of lengthening a signal component by inserting zeros between samples.

The filtering part of the reconstruction process also bears some discussion, because it is the choice of filters that is crucial in achieving perfect reconstruction of the original signal.

The downsampling of the signal components performed during the decomposition phase introduces a distortion called aliasing. It turns out that by carefully choosing filters for the decomposition and reconstruction phases that are closely related (but not identical), we can "cancel out" the effects of aliasing. The low- and high-pass decomposition filters (L and H), together with their associated reconstruction filters (L' and H'), form a system of what is called quadrature mirror filters.

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Looking at a signal's wavelet decomposition tree can yield valuable information.

Since the analysis process is iterative, in theory it can be continued indefinitely. In reality, the decomposition can proceed only until the individual details consist of a single sample or pixel. In practice, you'll select a suitable number of levels based on the nature of the signal, or on a suitable criterion such as entropy.

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